Exact Analysis of Shielded Microstrip Lines and Bilateral Fin Lines

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***Abstract*—An exact analysis is presented for shielded microstrip and bilateral fin lines. The method of analysis is based on function-theoretic approach to solve a set of functional equations in the Fourier transform domain, representing the independent excitation of LSE and LSM modes in the systems without strip or fin conductors. The solution is obtained in the form of highly convergent systems of algebraic equations, which allow the accurate calculation of fields and the electrical parameters of these lines at arbitrary frequencies.**

I. Introduction

MICROSTRIP LINES and bilateral fin lines belong to the general group of waveguiding structures composed of planar thin conductors, supported by dielectric layers and are frequently shielded by grounded conducting planes. The group also includes many other structures such as slot, coplanar, suspended strips, and various types of fin-line configurations. These structures have the main common advantage of ease of fabrication by printing techniques and therefore they find growing field of application as elements of microwave integrated circuits.

Among all these lines, suspended strip and microstrip lines are the best known and have been extensively used for quite a long time. Other lines, such as slot and fin-line configurations were recently investigated for use at higher microwave up to millimeter wave frequencies.

Microstrip lines have been thoroughly investigated by many authors both theoretically and experimentally [1]–[21]. However it seems that the problem has not yet been solved completely as evidenced by the considerable discrepancies between the published results remarked [21]. In fact most methods of analysis suffer from serious limitations and usually include assumptions that may lead to considerable uncertainty in the obtained results. Thus, most design calculations are still performed using the early quasi-static (TEM) results of Wheeler and others [1]–[4]. At relatively low frequencies, as long as the longitudinal field components have no significant values, the TEM-representation gives sufficiently accurate description of the propagation properties. At higher frequencies, due to the presence of the dielectric, wave effects become evident as revealed by the dispersion, change of wave impedance, and the presence of higher modes. These effects cannot be accounted for within the frame of the TEM theory. Therefore many authors tried to consider the complete problem of propagation of waves in such lines.

If purely numerical and qualitative methods are excluded, some general approaches can be distinguished. In some papers the problem has been attacked by transformation to systems of coupled integral equations, which were solved by different methods: moments, Galerkin, and variational [7], [16], [18]. The solution of these equations is not a simple task because of the complicated forms of the kernels and the fact that the kernels usually have singularities of the static type. To overcome these difficulties the spectral domain approach has been suggested [12], where integral equations are replaced by functional equations and the transforms of the kernels are sufficiently simple in form. Microstrip lines have also been treated by the singular equations method [8], where the singularities in the kernels are separated out. For the calculation of lines with finite thickness the field matching technique is specially useful [9], [11]. The method of transverse resonance was applied in [17] using the results of the problem of diffraction of plane wave over the edge of a semi-infinite parallel plate transmission line. A summary of the basic methods of analysis is given in [15].

Unilateral and bilateral fin lines were introduced as alternatives to microstrip lines for use at higher frequencies [23]–[25]. They were investigated by several authors using different approaches [26]–[29]. In view of the fact that fin lines are nearly always mounted in waveguides, there has been a general tendency to consider them as modified forms of ridged guides. This tendency reflected on the methods of analysis used, which are in fact very similar to those used for the treatment of ridged guides and waveguide discontinuities. The waveguiding properties of the gap between fins, regardless of the waveguide housing itself, were almost overlooked. In fact, fin lines are able to support guided waves even when the housing is removed altogether, as the fields are concentrated in the gap regions. In the following a method, based on modified Wiener–Hopf technique is applied for the analysis of shielded microstrip and bilateral fin lines without side walls. The formulation of the problem is exact and no assumptions were made during the solution. The high rate of convergence obtained allows essentially accurate determination of the electrical parameters of these lines.

II. Formulation of the Problem

Consider the dual structures shown on Fig. 1, comprising symmetrical strip and bilateral fin lines with two symmetrically located shields. The width of the strip con-

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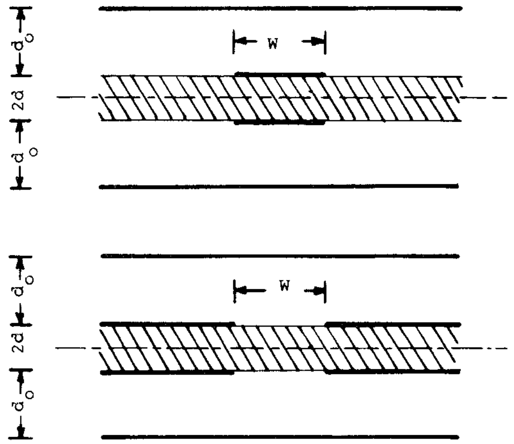

Fig. 1. Symmetrical stripline and bilateral fin-line configurations.

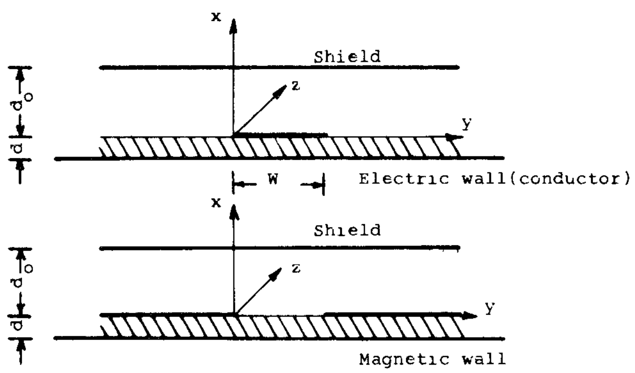

Fig. 2. Microstrip and bilateral fin-line models.

ductor or the gap between fins is denoted by W, the thickness of the dielectric is $2d$, and d_0 is the distance between the dielectric surface and the shield. The relative permittivity and the permeability of the dielectric are ϵ_r, μ_r, respectively, while those of free space are ϵ_0, μ_0. Fundamental mode fields of microstrip and bilateral fin lines correspond to electric and magnetic wall symmetry, respectively, in the strip and fin-line configurations of Fig. 1 with respect to the plane at the middle of the dielectric layer, leading to the basic models of Fig. 2. The Cartesian axes x, y, z are chosen as shown.

Considering the structure without the strip or fin conductors, which is in fact a dielectric loaded parallel plate waveguide, we assume a surface current distribution $J_y(y,z)$, $J_z(y,z)$ to flow over the dielectric surface. Surface currents will excite a field with electric field components tangential to the dielectric surface given by $E_y(0,y,z)$, $E_z(0,y,z)$.

The dependence of fields and currents on time t and the longitudinal coordinate z is taken in the form $e^{i(\gamma z-\omega t)}$, where γ is a real propagation constant and ω is the angular frequency. Fields and currents are expressed in terms of their Fourier transforms, defined for a function $f(y)$ as

$$\hat{f}(\alpha)=\int_{-\infty}^{\infty} f(y)e^{+i\alpha y}\,dy.$$

The general relation between the transforms of the tangential electric field components and the surface currents can be expressed as

$$\hat{E}_y(0,\alpha)=G_{11}(\alpha)\hat{J}_y(\alpha)+G_{12}(\alpha)\hat{J}_z(\alpha)$$
$$\hat{E}_z(0,\alpha)=G_{21}(\alpha)\hat{J}_y(\alpha)+G_{22}(\alpha)\hat{J}_z(\alpha) \qquad (1)$$

where G_{mn} are the transforms of the elements of the dyadic Green's function of the structure.

Functional relations (1) are equivalent to a pair of coupled integral equations in the space domain. Introducing the new variables U_1, U_2, F_1, F_2, as linear combinations of $\hat{J}_y$, $\hat{J}_z$, $\hat{E}_y$, $\hat{E}_z$ in the transform domain

$$U_1(\alpha)=-\alpha\hat{J}_y+\gamma\hat{J}_z \qquad F_1(\alpha)=-\alpha\hat{E}_y+\gamma\hat{E}_z$$
$$U_2(\alpha)=\gamma\hat{J}_y+\alpha\hat{J}_z \qquad F_2(\alpha)=\gamma\hat{E}_y+\alpha\hat{E}_z \qquad (2)$$

the set of equations (1) is diagonalized to the form

$$i\omega\epsilon_0\chi_1(\alpha)F_1(\alpha)=U_1(\alpha)$$
$$\chi_2(\alpha)F_2(\alpha)=i\omega\mu_0U_2(\alpha). \qquad (3)$$

Functions χ_1, χ_2 can be recognized as the transforms of inverse Green's functions for sources of pure LSM and LSE wave types, respectively, in the loaded guide without strips or fins. They have their zeroes rather than poles, coinciding with the propagation constants of these modes.

Explicit expressions for χ_1, χ_2 have the form

$$\chi_1(\alpha)=\frac{\coth R_0d_0}{R_0}+\epsilon_r\frac{\coth Rd}{R}$$
$$\chi_2(\alpha)=R_0\coth R_0d_0+\frac{1}{\mu_r}R\coth Rd$$

electric wall symmetry (microscope case)

or

$$\chi_1=\frac{\coth R_0d_0}{R_0}+\epsilon_r\frac{\tanh Rd}{R}$$
$$\chi_2=R_0\coth R_0d_0+\frac{1}{\mu_r}R\tanh Rd$$

for magnetic wall symmetry (fin-line case)

and

$$R_0=\sqrt{\alpha^2+\gamma^2-k_0^2}, \qquad k_0^2=\omega^2\epsilon_0\mu_0$$
$$R=\sqrt{\alpha^2+\gamma^2-k^2}, \qquad k^2=\epsilon_r\mu_rk_0^2.$$

Equations (3) are the starting point for the following discussion.

III. Fields in the Microstrip Line

The set of functional relations (3) can be used for the solution of the problem of propagation in the microstrip line. If the strip conductor is assumed to be thin and ideally conducting, then the following boundary conditions should hold:

$$\left.\begin{array}{llll} E_y(0,y)=0, & 0<y<W & J_y(y)=0, & y<0 \\ E_z(0,y)=0, & 0<y<W & J_z(y)=0, & y>W. \end{array}\right\} \quad (4)$$

Boundary conditions (4) will reflect on the properties of the functions F_1, F_2, U_1, U_2 as follows.

1) U_1, U_2 will be entire functions having algebraic behavior on the upper half of the α-plane.

2) F_1, F_2 can be expressed as

$$F_1(\alpha)=F_1^-(\alpha)\pm e^{i\alpha W}F_1^-(-\alpha)$$

and

$$F_2(\alpha)=F_2^-(\alpha)\mp e^{i\alpha W}F_2^-(-\alpha).$$

Functions F_1^-, F_2^- are regular in the lower half-plane and have algebraic behavior for large α. Upper signs refer to modes with symmetrical longitudinal current distribution on the strip, while lower signs to antisymmetrical modes.

Therefore, for the microstrip line the following functional equations can be written:

$$\frac{1}{i\omega\epsilon_0\chi_1}U_1(\alpha)=F_1^-(\alpha)\pm e^{i\alpha W}F_1^-(-\alpha)$$

$$\frac{i\omega\mu_0}{\chi_2}U_2(\alpha)=F_2^-(\alpha)\mp e^{i\alpha W}F_2(-\alpha). \quad (5)$$

Equations (5) allow solution using modified Wiener–Hopf technique [22].

It can be easily seen, that χ_1, χ_2 are meromorphic functions having their poles and zeroes lying symmetrically relative to the point of origin $\alpha=0$. Depending on frequency, some of them are real (when the medium is lossless), while all others are imaginary. Poles of χ_1, χ_2 coincide, except for the points defined by $R_0=0$, $R=0$ where χ_1 has poles while χ_2 is regular. These poles represent the propagation constants of waveguide modes in the strip region. Poles with positive imaginary part will be denoted by α_n while the zeroes of χ_1, χ_2 by ν_n, σ_n, respectively.

Factorizing χ_1, χ_2 to factors χ_1^+, χ_2^+ regular and having no zeroes on the upper half-plane, and χ_1^-, χ_2^- having the same properties on the lower half-plane, the relation between the plus and minus factors is given by

$$\chi_{1,2}^-(\alpha)=\chi_{1,2}^+(-\alpha).$$

Following the standard Wiener–Hopf procedure, multiplying equations (5) by χ_1^-, χ_2^-, separating the plus and minus parts and taking into account the asymptotic behavior of different terms as determined by the edge conditions we write

$$\chi_1^- F_1^-(\alpha)\pm\left[e^{i\alpha W}\chi_1^- F_1^-(-\alpha)\right]^-=P$$

$$\chi_2^- F_2^-(\alpha)\mp\left[e^{i\alpha W}\chi_2^- F_2^-(-\alpha)\right]^-=Q \quad (6)$$

where $[f]^\pm$ denotes the plus and minus parts of f and P, Q are some constants.

As the minus terms have only pole singularities on the upper half-plane, they can be expanded as follows:

$$\chi_1^-(\alpha)F_1^-(\alpha)\pm\sum_{m=0}^{\infty}e^{i\alpha_m W}F_1^-(-\alpha_m)\frac{\operatorname{Res}\chi_1^-(\alpha_m)}{\alpha-\alpha_m}=P$$

$$\chi_2^-(\alpha)F_2^-(\alpha)\mp\sum_{m=1}^{\infty}e^{i\alpha_m W}F_2^-(-\alpha_m)\frac{\operatorname{Res}\chi_2^-(\alpha_m)}{\alpha-\alpha_m}=Q. \quad (7)$$

Equations (7) can be solved either by direct iteration or by transformation to systems of linear algebraic equations for the unknown coefficients $F_{1,2}^-(-\alpha_m)$. The presence of the exponential factors $e^{i\alpha_m W}$, where all α_m except α_0 are imaginary, guarantees high rate of convergence of both methods.

Introducing the notation

$$PA_n=\chi_1^-(-\alpha_n)F_1^-(-\alpha_n)$$

$$QB_n=\chi_2^-(-\alpha_n)F_2^-(-\alpha_n)$$

(7) can be transformed to the following inhomogeneous systems of equations for A_n, B_n:

$$A_n=1+\sum_{m=0}^{\infty}\frac{\xi_m}{\alpha_n+\alpha_m}A_m, \qquad n=0,1,\cdots$$

$$B_n=1+\sum_{m=1}^{\infty}\frac{\zeta_m}{\alpha_n+\alpha_m}B_m, \qquad n=1,2,\cdots. \quad (8)$$

Coefficients ξ_n, ζ_n are given by

$$\xi_n=\frac{\operatorname{Res}\chi_1^-(\alpha_n)}{\chi_1^-(-\alpha_n)}e^{i\alpha_n W}$$

$$\zeta_n=-\frac{\operatorname{Res}\chi_2^-(\alpha_n)}{\chi_2^-(-\alpha_n)}e^{i\alpha_n W}.$$

Signs in (8) are taken to correspond to the fundamental symmetrical microstrip mode.

When the strip width is not too small, systems (8) are rapidly convergent and can be effectively solved by iteration techniques to practically any required degree of accuracy. Once A_n, B_n are determined, functions F_1^-, F_2^- can be obtained through the expressions

$$F_1^-(\alpha)=\frac{P}{\chi_1^-}\left\{1-\sum_{n=0}^{\infty}\frac{\xi_n}{\alpha-\alpha_n}A_n\right\}$$

$$F_2^-(\alpha)=\frac{Q}{\chi_2^-}\left\{1-\sum_{n=1}^{\infty}\frac{\zeta_n}{\alpha-\alpha_n}B_n\right\}. \quad (9)$$

It should be noted, that until this point, fields of LSM and LSE types were treated quite independently. This is the main advantage of the introduction of the variables F_1, F_2, U_1, U_2. However, fields in microstrip lines should be of the hybrid type and LSM, LSE fields are necessarily coupled. This coupling is actually present as P and Q have to satisfy certain relations so as to achieve physically proper field behavior.

Once F_1, F_2, U_1, U_2 are determined from the solution of (8), the physical variables $\hat{J}_y$, $\hat{J}_z$, $\hat{E}_y$, $\hat{E}_z$ can be retrieved through transformations inverse to (2). These inverse linear transformations are singular at values of α given by $\alpha = \pm i\gamma$. Since $\hat{J}_y$, $\hat{J}_z$ are entire functions, the constants P and Q must be chosen such that these singularities are cancelled out. This is achieved when U_1 and U_2 satisfy the relations

$$U_1(\pm i\gamma) \pm iU_2(\pm i\gamma) = 0$$

which can be shown to be equivalent to

$$F_1^-(\pm i\gamma) \pm iF_2^-(\pm i\gamma) = 0. \tag{10}$$

Condition (10) leads to a set of two homogeneous linear equations in P and Q. For nonvanishing fields the determinant must be equal to zero. This determines the possible values of the propagation constant γ and the ratio P/Q which can be looked upon as a measure for LSM–LSE field coupling.

The wave impedance of the microstrip line, which is taken to be the ratio of quasistatic voltage at the strip center to the total longitudinal current flowing on the strip, can be expressed directly through the function F_1^-:

$$Z_0 = \frac{1}{2\alpha_0} \frac{\gamma}{\omega\epsilon_0} \frac{F_1^-(-\alpha_0)}{F_1^-(0)} \frac{e^{i\alpha_0 W/2}}{\chi_1(0)}$$

where α_0 is the zero-order pole of χ_1 given by

$$\alpha_0 = \sqrt{k^2 - \gamma^2}.$$

IV. Wave Propagation in Bilateral Fin lines

In this case the presence of fins will impose boundary conditions dual to (4):

$$\begin{aligned} J_y(y) &= 0, \quad 0 < y < W \\ J_z(y) &= 0, \quad 0 < y < W \\ E_y(0,y) &= 0, \quad y < 0, \quad y > W \\ E_z(0,y) &= 0, \quad y < 0, \quad y > W. \end{aligned} \tag{11}$$

To satisfy boundary conditions (11) the following properties must be prescribed to the F and U functions: 1) F_1, F_2 are entire functions having algebraic behavior on the upper half-plane; 2) U_1, U_2 for fin-line modes with antisymmetrical longitudinal currents should be represented in the form

$$U_1(\alpha) = U_1^-(\alpha) + e^{i\alpha W} U_1^-(-\alpha)$$

and

$$U_2(\alpha) = U_2^-(\alpha) - e^{i\alpha W} U_2^-(-\alpha)$$

where U_1^-, U_2^- are functions, regular on the lower half-plane.

The set of functional equations for the bilateral fin-line problem can be written

$$\begin{aligned} i\omega\epsilon_0 \chi_1(\alpha) F_1(\alpha) &= U_1^-(\alpha) + e^{i\alpha W} U_1^-(-\alpha) \\ \frac{1}{i\omega\mu_0} \chi_2(\alpha) F_2(\alpha) &= U_2^-(\alpha) - e^{i\alpha W} U_2^-(-\alpha) \end{aligned} \tag{12}$$

where χ_1, χ_2 are the inverse Green's functions for fields with magnetic wall symmetry.

Following the same procedure used for the microstrip problem, functions U_1^-, U_2^- can be represented in the form

$$U_1^-(\alpha) = P\chi_1^-(\alpha)\left\{1 - \sum_{n=1}^{\infty} \frac{\xi_n}{\alpha - \nu_n} A_n\right\}$$

and

$$U_2^-(\alpha) = Q\chi_2^-(\alpha)\left\{1 - \sum_{n=1}^{\infty} \frac{\zeta_n}{\alpha - \sigma_n} B_n\right\}$$

where A_n, B_n are determined from the following systems of equations

$$\begin{aligned} A_n &= 1 + \sum_{m=1}^{\infty} \frac{\xi_m}{\nu_n + \nu_m} A_m, \quad n = 1, 2, \cdots \\ B_n &= 1 + \sum_{m=1}^{\infty} \frac{\zeta_m}{\sigma_n + \sigma_m} B_m, \quad n = 1, 2, \cdots. \end{aligned} \tag{13}$$

P and Q are yet undetermined constants and the coefficients ξ_n, ζ_m are given by

$$\xi_n = -\frac{[\chi_1^+(\nu_n)]^2}{\chi_1'(\nu_n)} e^{i\nu_n W}$$

$$\zeta_n = \frac{[\chi_2^+(\sigma_n)]^2}{\chi_2'(\sigma_n)} e^{i\sigma_n W}.$$

ν_n, σ_n are the roots of χ_1, χ_2 lying on the upper half-plane, most of which are imaginary. Therefore except for very narrow-gap fin lines, systems (13) are highly convergent due to the exponential factors in ξ_n, ζ_n and A_n, B_n can be easily calculated to any required degree of accuracy. The constants P and Q, as in the case of microstrip lines, are determined by the analyticity of the transforms of surface currents and tangential electric field components. To cancel the singularities introduced by the inverse transforms from $F_{1,2}$ and $U_{1,2}$ to $\hat{J}_y, \hat{J}_z, \hat{E}_y, \hat{E}_z$ the following equivalent conditions have to be satisfied:

$$F_1(\pm i\gamma) \pm iF_2(\pm i\gamma) = 0$$

$$U_1^-(\pm i\gamma) \pm iU_2^-(\pm i\gamma) = 0.$$

Therefore P and Q satisfy two homogeneous linear equations, the simultaneity of which determines the possible values of γ and the ratio P/Q, indicating the field coupling. The impedance of the bilateral fin line, which will be taken as the ratio of the quasi-static gap voltage to the total longitudinal current on one of the fins, can be expressed directly in terms of the U functions as

$$Z_0 = \frac{2\omega\mu_0 i}{\chi_2(0)} \frac{U_2^-(0)}{U_1^-(0)}.$$

V. Physical Interpretation and Numerical Results

The properties of propagation in microstrip and bilateral fin lines as determined by the solution of the sets (8), (13) are dependent mainly on the behavior of poles and roots of inverse Green's functions χ_1, χ_2. It is easy to show that poles and zeroes are either real or purely imaginary, i.e., the squares of their values are always real. Moreover, they form interleaving sequences, thus between any two poles one root can be found and vice versa.

Considering the case of the microstrip line, poles compose two sets:

$$\alpha_n \begin{cases} a_n = \sqrt{k^2 - \gamma^2 - (n\pi/d)^2} \\ b_n = \sqrt{k_0^2 - \gamma^2 - (n\pi/d_0)^2} \end{cases}$$

where $n=0,1,2,\cdots$ for χ_1 and $n=1,2,\cdots$ for χ_2. These poles correspond to the waveguide modes in the strip region $0<y<W$, between the strip and the base conductor and the strip and the shield. Zeroes correspond to the loaded guide modes in the external regions $y<0$, $y>W$. Poles and zeroes are arranged in the following manner:

$$(a_n^2, b_n^2, \nu_n^2) < \nu_1^2 < a_0^2$$

$$(a_n^2, b_n^2, \sigma_n^2) < \sigma_1^2 < b_0^2 < a_0^2$$

where (a_n^2, b_n^2, ν_n^2), $(a_n^2, b_n^2, \sigma_n^2)$ denote the sets of all other poles and roots.

For unattenuating propagation, all zeroes must be imaginary, otherwise power will be lost by radiation in the broad-side directions. Therefore considering the pole–root relation the only real pole is α_0, corresponding to the lowest order TEM parallel plate guide mode in the region beneath the strip. All other modes are decaying in the y-direction away from the edge positions, where they are excited. Therefore it can be concluded, that the field in the microstrip propagates essentially in a multiple reflection mode as a result of total reflection of TEM-waves in the strip-ground plane region propagating at angles $\pm\psi$, $\psi = \cos^{-1}(\gamma/k)$ to the z-axis from strip edges. In case of narrow strips or at low frequencies this picture is distorted by the coupling of the strip edges. In case of wide strips or at high enough frequencies this coupling becomes less significant and only the lowest order TEM-mode in (8) with $n=0$ should be considered. This has been verified actually by direct computation.

The case of bilateral fin lines is somewhat different due to the specific zero-pole behavior. Thus, propagation of unattenuating waves in bilateral fin lines without side walls necessitates that all poles of χ_1, χ_2 should be imaginary. Otherwise the excitation of waveguide modes in the regions $y<0$, $y>W$ would render the main wave leaky. The poles form the sets

$$\alpha_n \begin{cases} \sqrt{k^2 - \gamma^2 - \left[\left(n - \frac{1}{2}\right)\pi/d\right]^2} \\ \sqrt{k_0^2 - \gamma^2 - (n\pi/d_0)^2} \end{cases}.$$

Therefore the allowed values of γ are limited to the range

$$k^2 > \gamma^2 > \max\left\{k_0^2, k^2 - (\pi/2d)^2\right\}.$$

For this range of γ all propagation constants of LSM and LSE modes in the gap region are imaginary, except for the lowest order LSE mode corresponding to the first root of χ_2. This root, denoted by σ_1, can be either real or imaginary. Depending on the value of σ_1, two modes of propagation in fin lines can be distinguished. When σ_1 is imaginary the field in the gap region has a quasi-static character and all coefficients ξ_n, ζ_n are real. When σ_1 is real, the field propagates in a waveguide mode, guided by multiple reflections of the surface wave from fin edges, where conditions of total reflection exist as all waveguide modes are evanescent. Computations have shown, that the two modes are possible. Quasi-static mode dominates at low frequencies while the waveguide mode is dominant at high enough frequencies.

Calculations revealed a curious behavior of the dispersion curves of fin lines at different gap widths in the transition region between quasi-static and waveguide modes. It was found that these curves, regardless of the gap width, intersect at a common point on the line representing the dispersion characteristics of the surface wave mode corresponding to σ_1. This can be explained by the fact that the effect of width of the gap on the dispersion characteristics is different in the two regions. Thus in the quasi-static mode smaller gaps tend to lower the phase velocity due to field concentration in the dielectric. In the waveguide mode this effect is reversed as wider gaps tend to decrease the phase velocity towards the value for the free surface wave velocity. This effect is analogous to the effect of width in rectangular guides. Therefore the family of dispersion curves at different widths should have an intersection point where the effect is reversed.

Following this analysis it must be remarked that the bilateral fin line is sensitive to geometrical imperfections violating the symmetry of the field, e.g., relative displacement of the gaps. In this case the fundamental TEM mode in the dielectric filled waveguide between the fins will be excited, leading to loss of power in the side directions in the fin lines without walls or to strong coupling to the walls if they are present.

Computations were performed for microstrip lines with high and low ϵ_r dielectrics $\epsilon_r = 9.7$, 2.32 for different strip widths and the dependence of effective dielectric constant ϵ_{eff} and line impedance on frequency is illustrated by Figs. 3 and 4, where the approximate results obtained by using only one term in (8) are also shown as dashed lines. Coincidence of the approximate and exact calculations over a considerable part of the frequency interval indicates the high rate of convergence of the infinite sets (8), specially at high frequencies. Moreover, this confirms the suggestion about the nature of propagation in microstrip lines.

Fig. 5 shows the dependence of the P/Q ratio upon frequency. At low frequencies LSM field component dominates, justifying the quasi-static TEM theory at these frequencies, at least for lines with wide strips.

Results of computation of the dispersion characteristics and wave impedance as functions of frequency are shown

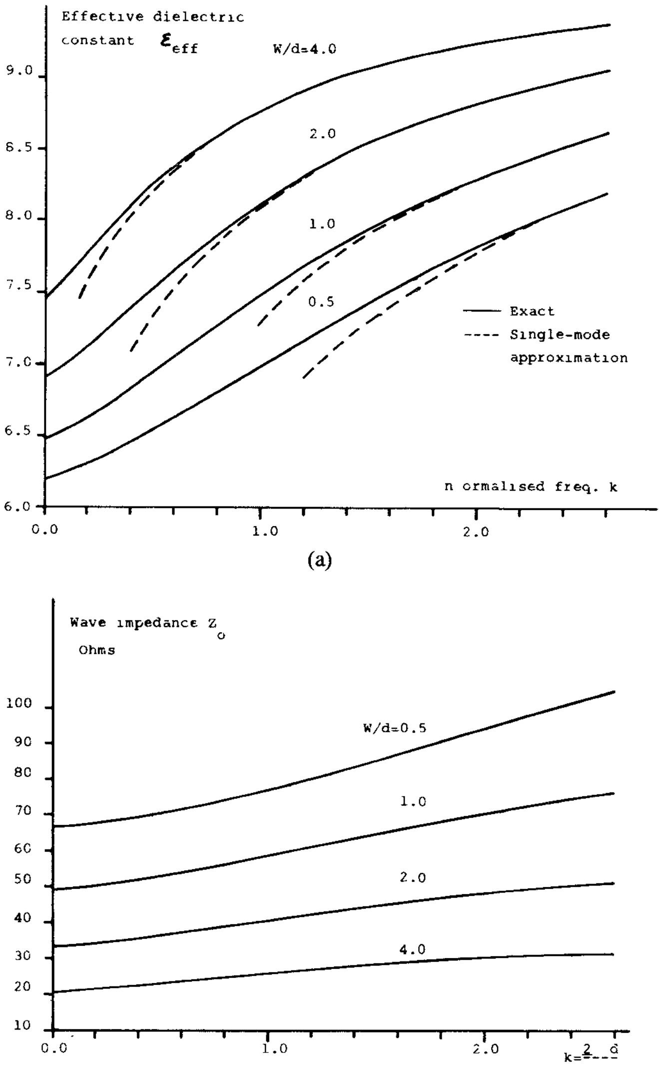

Fig. 3. (a) Dispersion characteristics of microstrip line. $\epsilon_r=9.7$, $\mu_r=1.0$, $d_0/d=10.0$. (b) Variation of microstrip line impedance with frequency. $\epsilon_r=9.7$, $\mu_r=1.0$, $d_0/d=10.0$.

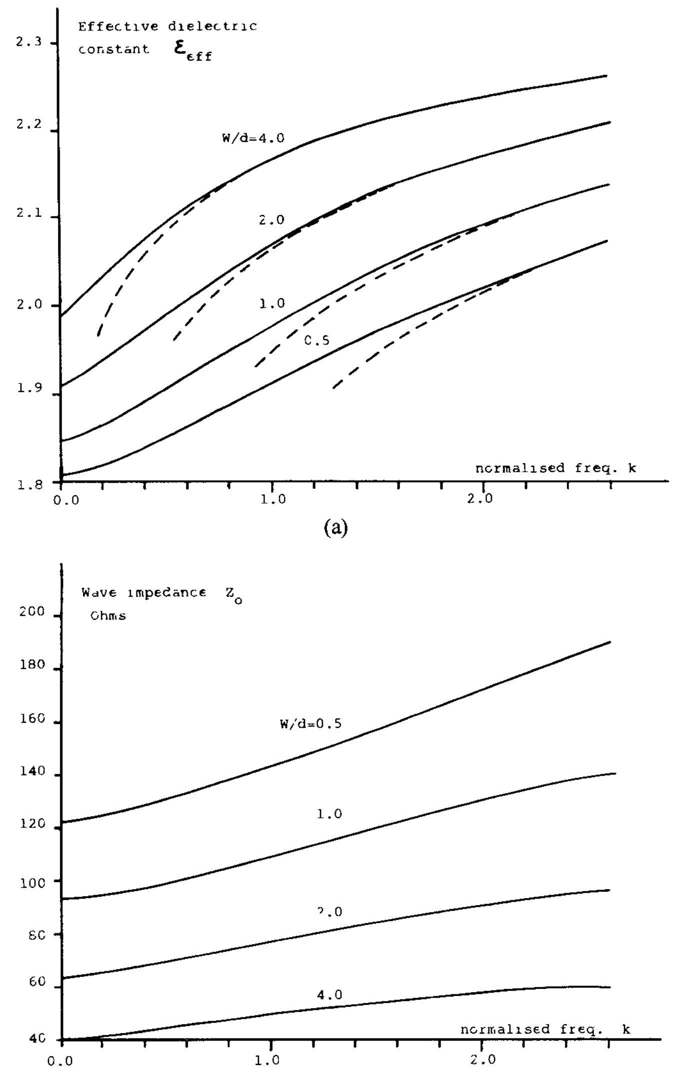

Fig. 4. (a) Dispersion characteristics of microstrip line. $\epsilon_r=2.32$, $\mu_r=1.0$, $d_0/d=10.0$. (b) Variation of microstrip impedance with frequency, $\epsilon_r=2.32$, $\mu_r=1.0$, $d_0/d=10.0$.

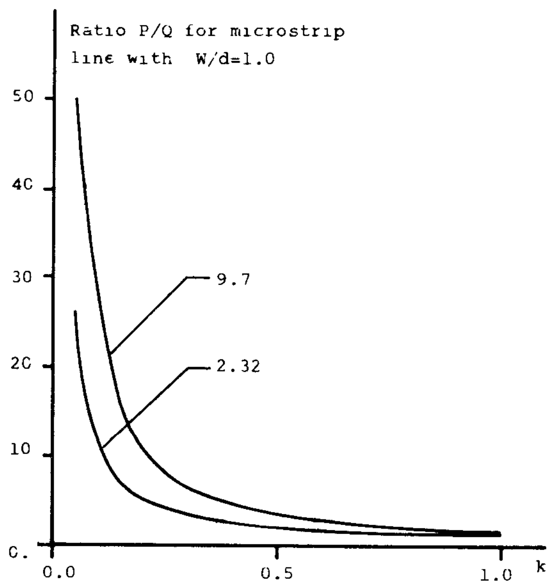

Fig. 5. Dependence of the ratio P/Q (measure of LSM–LSE field coupling) on frequency for microstrip lines.

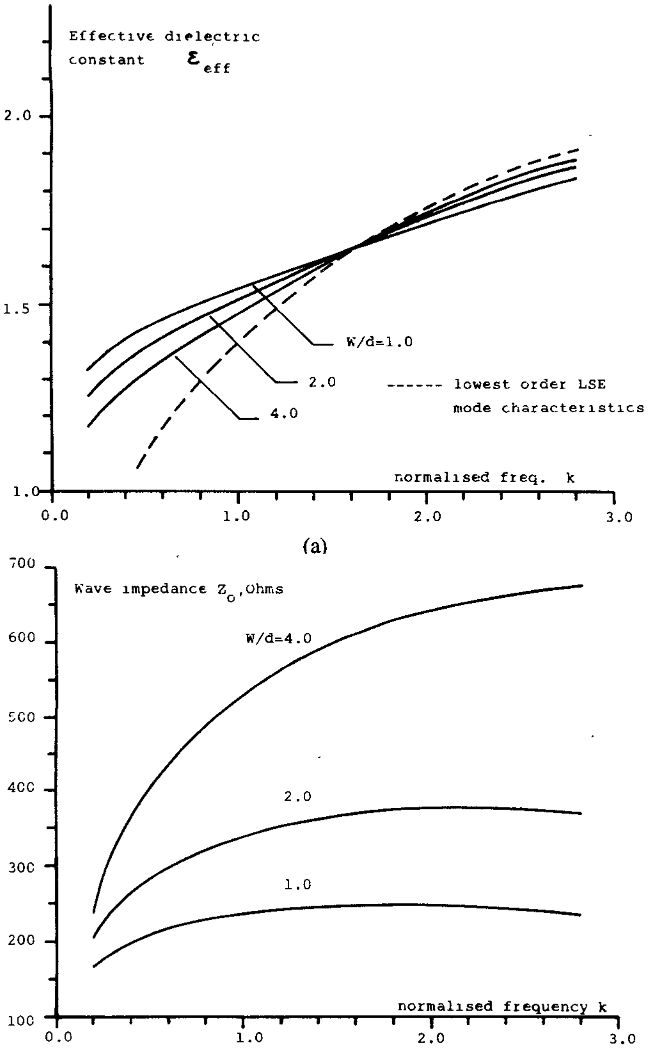

Fig. 6. (a) Dispersion characteristics of bilateral fin line. $\epsilon_r = 2.22$, $\mu_r = 1.0$, $d_0/d = 10.0$. (b) Wave impedance of bilateral fin line versus frequency. $\epsilon_r = 2.22$, $\mu_r = 1.0$, $d_0/d = 10.0$.

on Fig. 6, for bilateral fin lines with different gap widths on low dielectric substrate of the type usually used for these lines. The dispersion curves clearly show the peculiar behavior of these lines at the transition region.

In all graphs the normalized frequency variable

$$k = (2\pi d/\lambda_0)\sqrt{\epsilon_r}$$

where λ_0 is the free space wavelength, is used.

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